Modelling Catchment Pollution from Intensive Agricultural Practices

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Abstract

Many agricultural projects begin by making sense of data, which has been collected in a sometimes-arbitrary fashion without some real goal or purpose, and most likely the reason for its collection is not very clear. It is important to know what is wanted and what is needed to understand before data is assembled. Therefore having a pre-determined model is important. In many circumstances quite basic statistical techniques are used by practitioners, who are often unfamiliar with the recent agricultural systems-techniques now available internationally to understand, use and process this data. Often few attempts are made to track "cause- to effect", which is essential especially when trying to advise resource implications in places like New Zealand. This was also behind the approach used in the remediation of the Tha Chin river in Bangkok, Thailand, and this is described in Pimpunchat *et al* [1,2].

In this exposition it is intended to highlight the use of stochastic processes as a way of quantifying the dynamic relationships of sequences of random events. Stochastic models play an important role in elucidating many areas of the agricultural and ecological sciences which, for countries like New Zealand, are important and highly valued. They can be used to analyse the variability inherent in these processes and give some precision to their understanding. See Pleasants *et al* [3].

Here emphasis is given to transforming a Stochastic Ito-type process model into a more familiar Fokker-Planck equation with drift to enable data to be usefully interpreted and to give stochastic predictions to be made for agricultural practice and predictions for use in catchment areas. For an introduction to these methods, see for example, Rinsky and Karlin [4].

Intensive farming practice leads to cumulative effects, so the "run-off" adds sequentially to give a long-term prediction. The Fokker-Planck equation is akin to the more familiar advection-dispersion equation which is firstly a non-autonomous linear partial differential for the evolving probability density function of the degree of pollution. The drift-term enables it to be transformed into a more familiar equation which is tractable analytically. This is of value in understanding "downstream effects" in the catchment models.

Here we use the proportion of pasture area incorporating the stocking rate and focus on the degree of pollution y. $0 \leq y \leq 1$ (= the clean portion). This then starts to be polluted in a simple fashion as the rate-of-decay equation with deposition rate α per day

$$dy/dt = -\alpha y, y(0) = 1.$$

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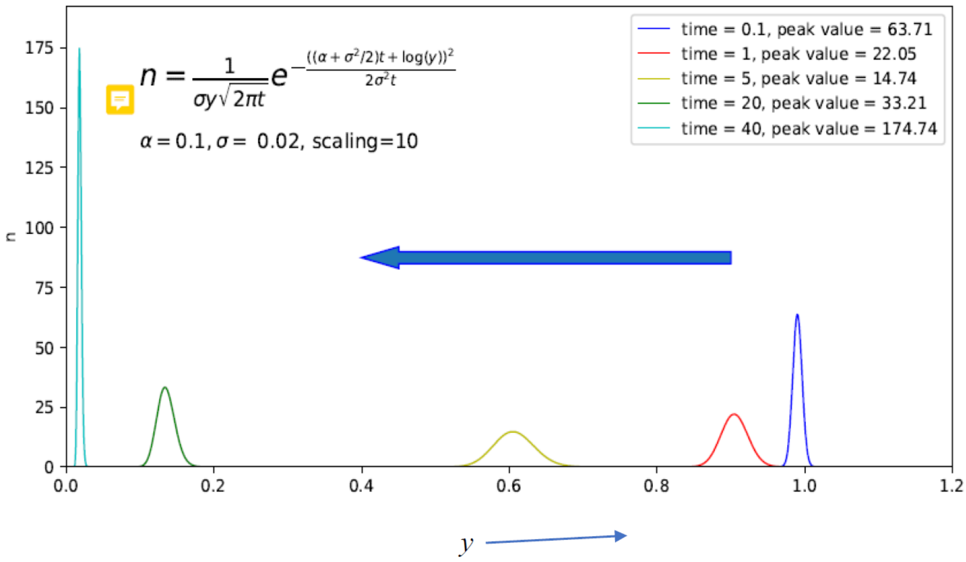
Turning this into a Markov process gives the stochastic differential equation

$$dY = -\alpha Y\,dt + \sigma Y\;dW,\;\; Y(0) = 1.$$

Here $dW^2 = dt$ as with any Wiener process, and σ is the standard deviation of the deposition rate α. Using the Fokker-Plank version of this, we get the dispersion-drift equation for the evolving probability density function $n(y,t)$

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial y}(\alpha\;y\;n) + \frac{1}{2}\frac{\partial^2}{\partial y^2}(\sigma^2 y^2 n),\;\; t > 0\;\; 0 < y < 1,$$

and $n(y,0) = \delta(y-1)$ (all clean), where δ is the Dirac-delta function. The solution is shown to be a logarithmic-normal distribution, with a distribution shown in the illustrative Figure below: The arrow of time [days] is to the left. Notice the long-time effect is, as expected, also a Dirac-delta function.



This model extends easily to multi-patches where there are equivalent conditional probability-density functions for the distribution of many patches. This enables a joint distribution to be found, expected to be a variation of the familiar logarithmic normal. Simulations match this.

Current work is aimed at using this to underpin a multi-component catchment model which will embed this into a multi-purpose tool to underpin decision-making and to inform sustainable farming practice.

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References

[1] B Pimpunchat, WL Sweatman, GC Wake, W Triampon, A Parshotam. *A mathematical model for pollution in a river and its remediation by aeration (62 citations)*. Applied Mathematics Letters 22 (3), 304-308 2007.

[2] B Pimpunchat, WL Sweatman, W Triampo, GC Wake, A Parshotam. *Modelling river pollution and removal by aeration (16 citations)* MODSIM 2007 International Congress on Modelling and Simulation. Land, Water, 2007.

[3] AB Pleasants, PR Shorten, GC Wake. *The distribution of urine deposited on a pasture from grazing animals (56 citations)* The Journal of Agricultural Science 145 (1), 81-86, 2007.

[4] Mark A Pinsky Samuel Karlin. *An introduction to stochastic modelling (fourth edition)* Academic Press,Elsevier 2011.